A Phase-Field Fracture Model for Crevasse Behavior in Glaciers

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Introduction and methodology of our model:

Phase-field in fracture mechanics:

- A competitive approach for realistic problems (e.g., crack branching)
- Treats the crack as a second phase
- Uses gradient terms to smear out the crack faces (for standard numerical methods)

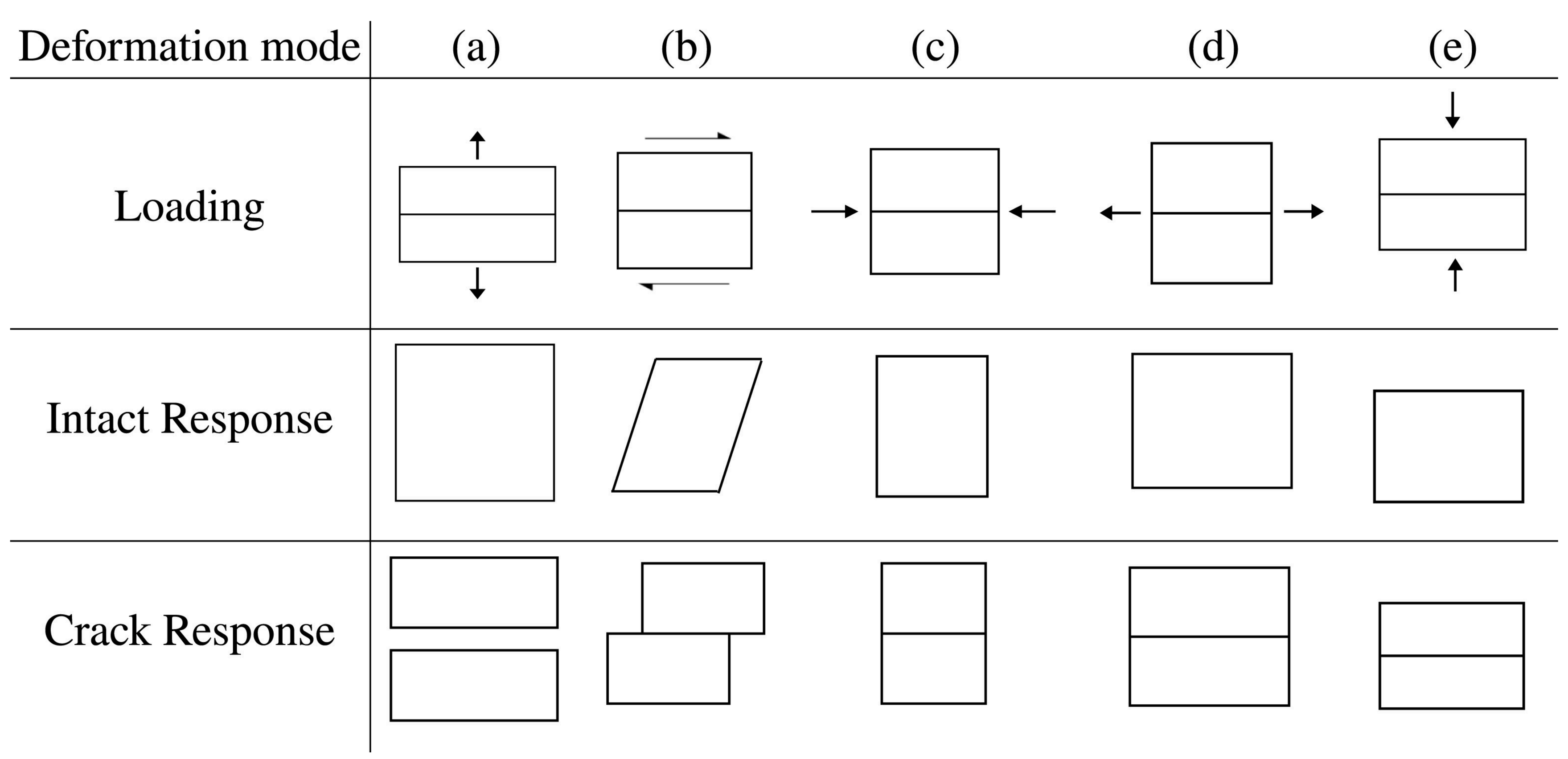
Shortcomings of the existing phase-field models:

- Inability to accurately model crack faces closing / shearing

Our proposed crack energy model:

$$\int_\Omega \left((1-|\boldsymbol{d}|)^2 W(\nabla \boldsymbol{y}) + \left(1-(1-|\boldsymbol{d}|)^2\right) W_{\mathrm{d}}(\nabla \boldsymbol{y}, \boldsymbol{n})\right) \mathrm{d}V_{\boldsymbol{x}} + G_c \int_\Omega \left(\frac{|\boldsymbol{d}|^2}{2\epsilon} + \frac{\epsilon}{2}|\nabla \boldsymbol{d}|^2\right) \mathrm{d}V_{\boldsymbol{x}}$$

- Accounts for the direction of the crack
- Classifies the deformation mode using **QR** decomposition
- Provides the appropriate material response based on type of deformation



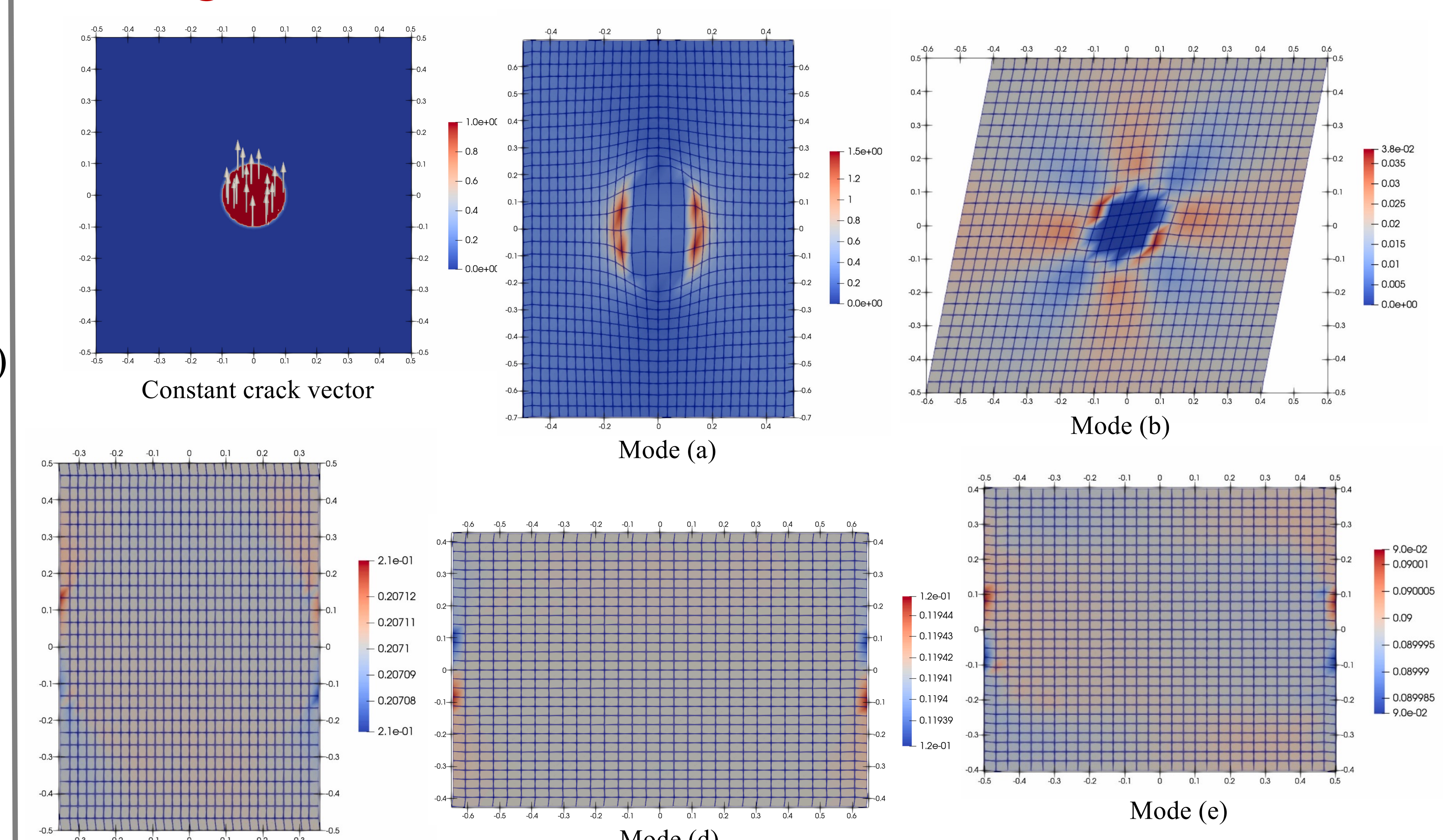
NeoHookean material utilized, enabling simulation in the large deformations

Methods of simulation used in this work:

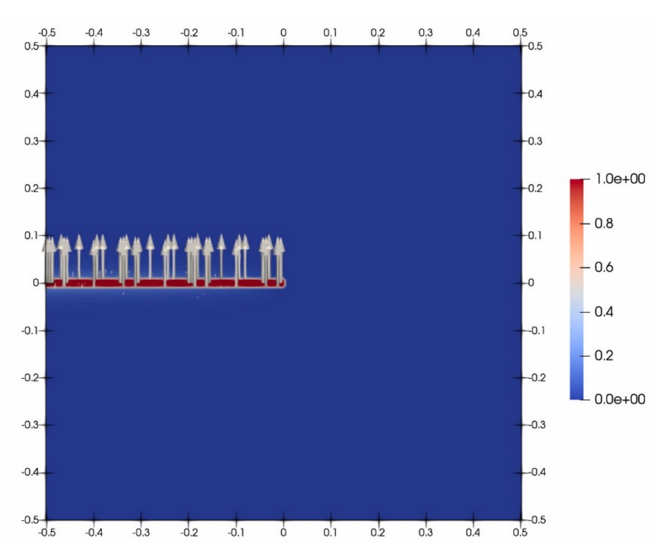
Dolfin-Adjoint library: To minimize the energy WRT displacement for constant cracks

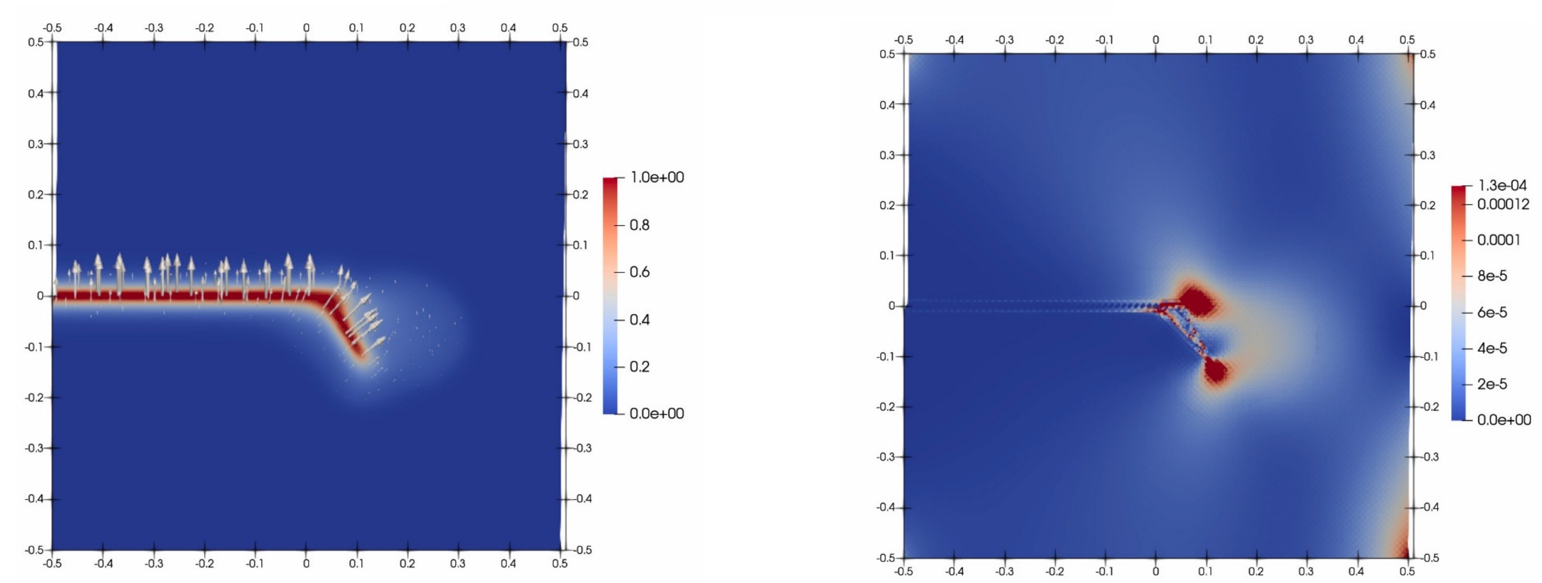
FEniCS library: To minimize energy WRT crack vector and displacement for crack growth

Modeling different deformation modes for a constant crack:

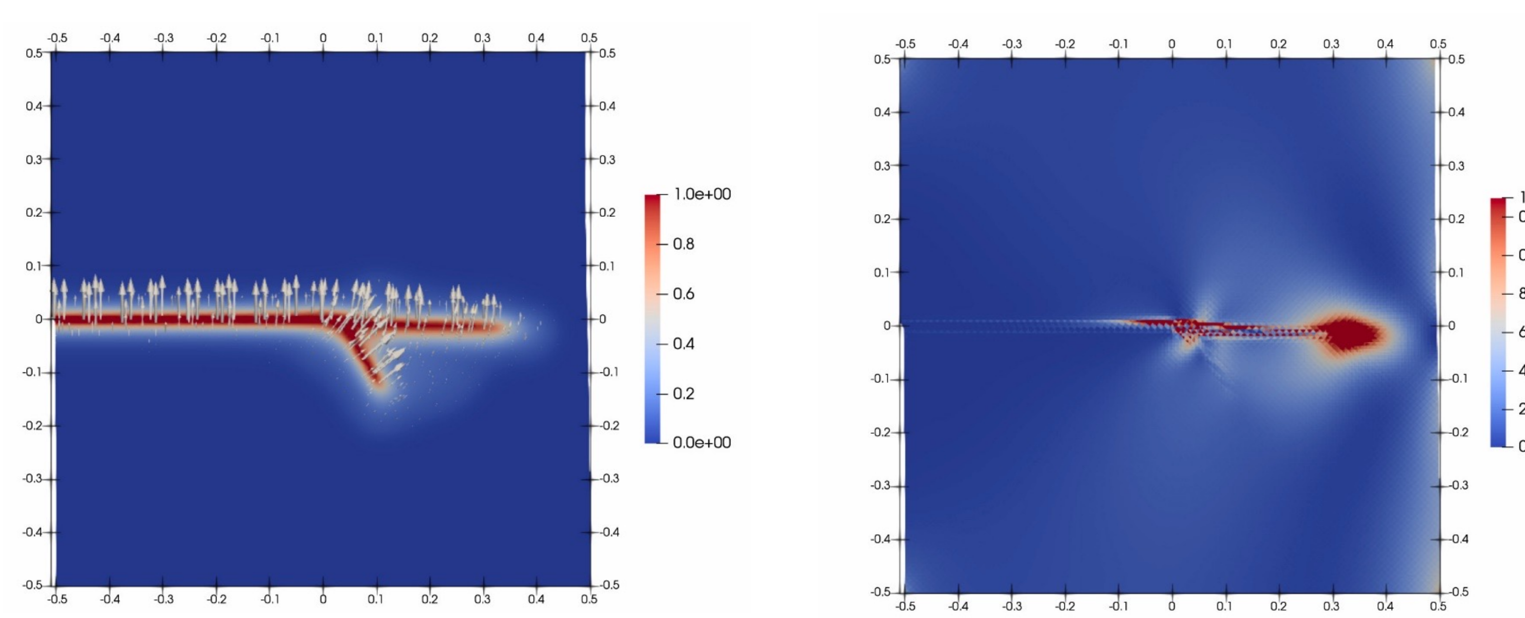


Crack growth under cyclic shear loading:



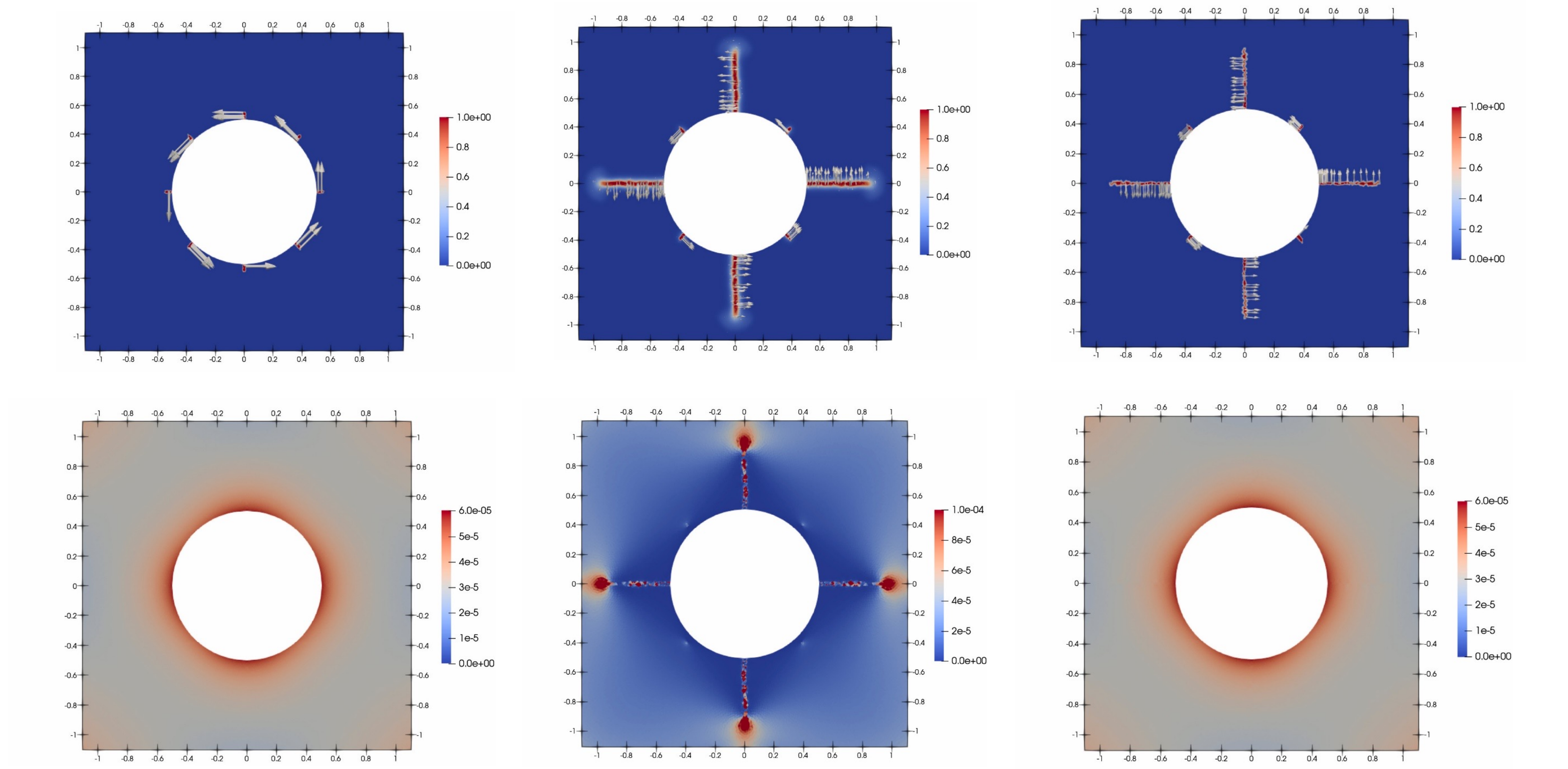


Crack growth under shear to the right (left Fig) and the elastic energy associated with that (right Fig)



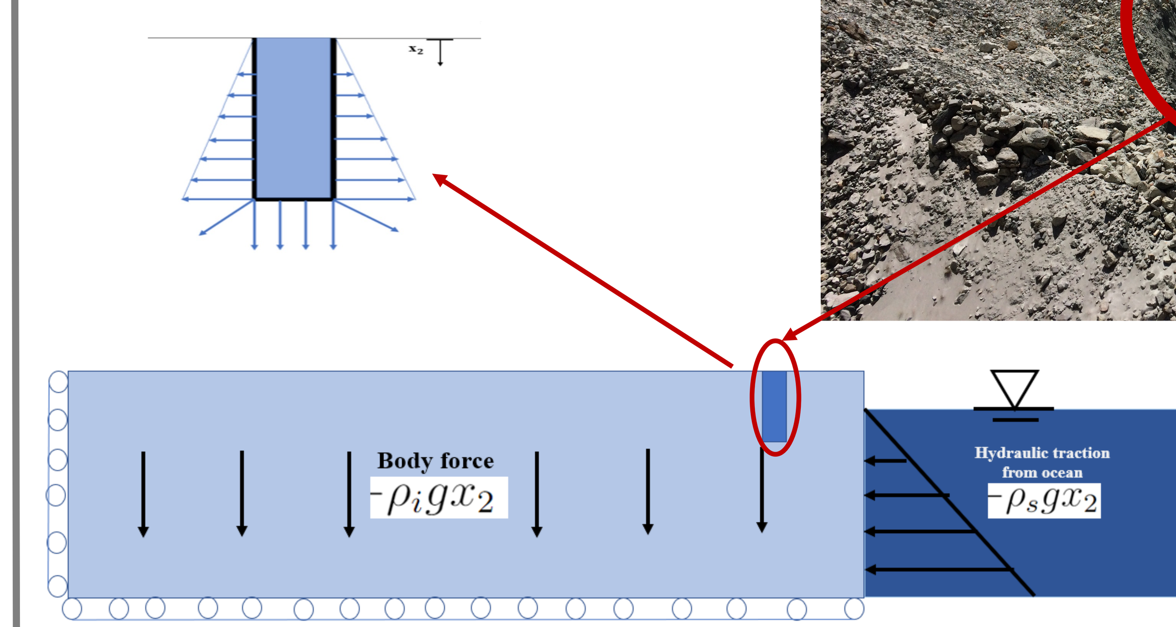
Crack growth under shear to the left (left Fig) and the elastic energy associated with that (right Fig)

Cyclic loading:



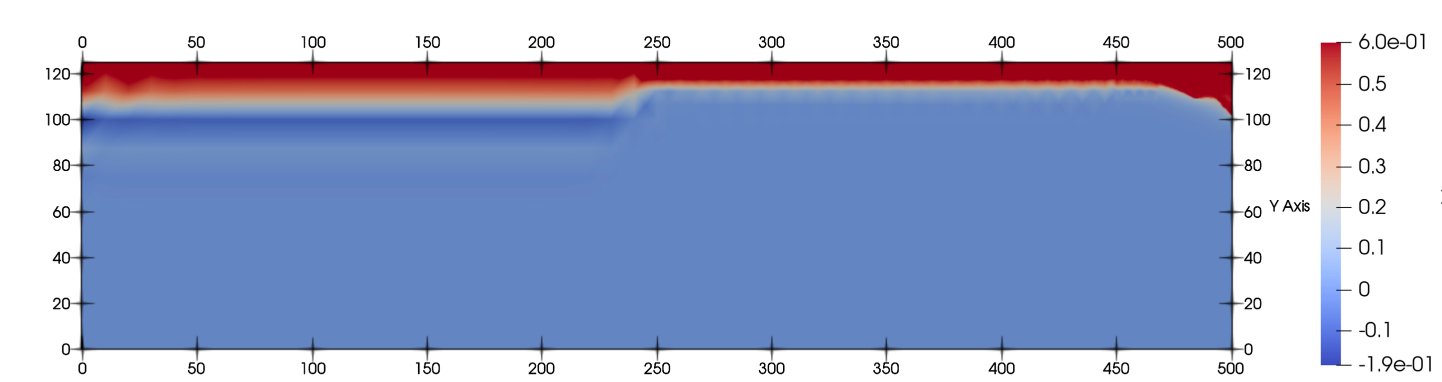
Top row: Initial cracks under compression (left Fig), followed by expansion (middle Fig.), and second cycle of compression (right Fig.).
Figures at the bottom row show the elastic energy corresponding to each load at the top row.

Crevasse growth in glaciers:



Previous models do not consider crack closure.

Modeling crevasse nucleation and propagation:



Stress-induced melting:

- An important factor in ice calving.
- Previous studies for modeling crevasse growth did not consider it.

Modeling realistic glaciers by surrogate modeling:

- Replace the detailed damage mechanics model with a surrogate
- Important for study of glaciers at field scales
- Use the results of these calculations as input for glacier models

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